Strongly-interacting phases in heavy-ion collisions and neutron stars

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with
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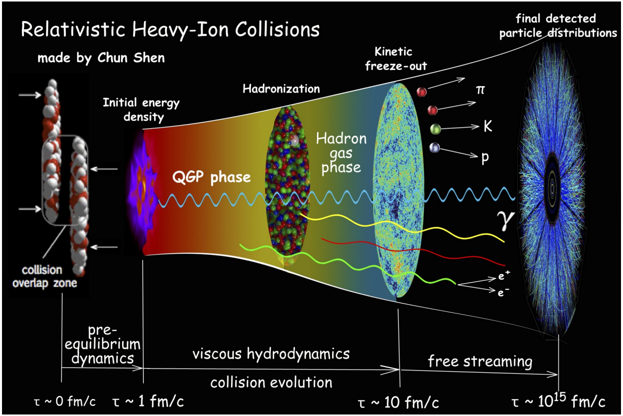
&
data from the STAR Collaboration

April 7th, 2023

HIC & QCD



[Chun Shen, The Ohio State University]

- HIC's to probe QCD phase structure & CEP
- Pheno. model → EoS → Hydro. sim. of eqlbm. stage → phase structure
- Exp.-data analysis → femtoscopy → source-geometry → phase structure

PART I: PHENOMENOLOGY

Ideal, relativistic hydrodynamics

- Macroscopic description of ideal fluid requires conserved quantities
- Ideal fluid: a continuous system of infinitesimal volume elements, each of which are assumed to be very close to thermodynamic equilibrium
- Conservation laws: $\nabla_\mu T^{\mu\nu}_{(0)} = 0$, $\partial_\mu N^\mu_{(0)} = 0$
- Fields: ε, P, n and u^μ correspond to 6 degrees-of-freedom
- Equations of motion:

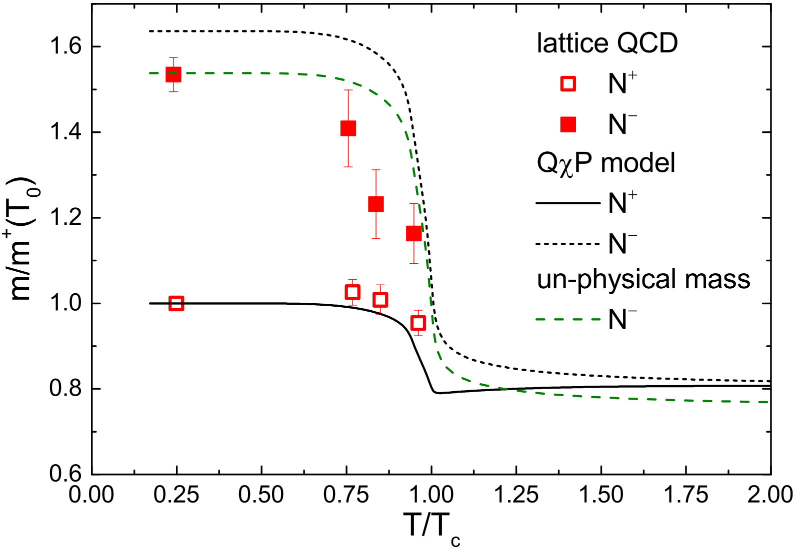
$$\begin{aligned} D\varepsilon + (\varepsilon + P)\theta_\mu u^\mu &= 0 \\ (\varepsilon + P)Du^\alpha + c_s^2\theta^\alpha\varepsilon &= 0 \\ Dn + n\partial_\mu u^\mu &= 0 \\ c_s^2(\varepsilon) - \frac{\partial P(\varepsilon)}{\partial\varepsilon} &= 0 \end{aligned}$$

- Equation-of-State: $P \equiv P(n, \varepsilon)$ from thermodynamic model based on microscopic theory of strong interactions

The QχP model

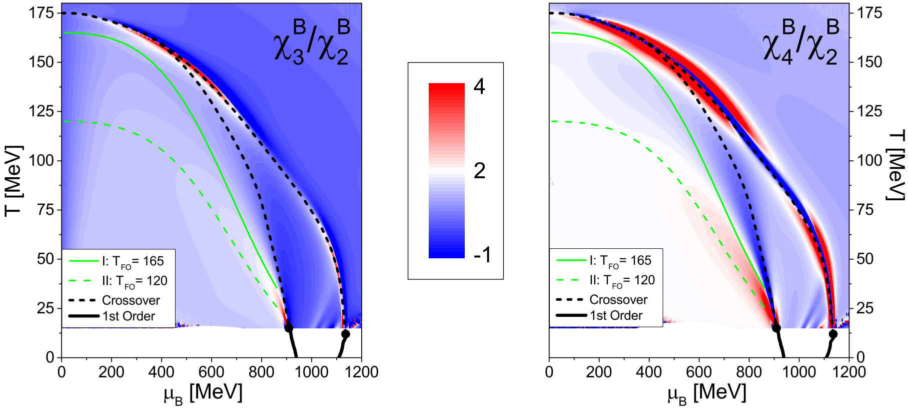
- Description:
  - Flavour SU(3) extension of non-linear representation of σ-ω model
  - Grand-canonical, thermodynamic model
  - Effective mass of baryons:
    $$m^*_{i\pm} = \sqrt{\left[(g^{(1)}_{\sigma i}\sigma + g^{(1)}_{\zeta i}\zeta)^2 + (m_0 + n_s m_s)^2\right]} \pm g^{(2)}_{\sigma i}\sigma \pm g^{(2)}_{\zeta i}\zeta$$
  - Order-parameter for chiral transitions: $\sigma = \langle\overline{\psi}\psi\rangle$
  - Order-parameter for deconfinement: Polyakov loop, ϕ
  - Hadrons removed, post deconfinement, with excluded-volumes
- Objectives:
  - Qualitative agreement with lattice-QCD predictions
  - χ^n_B near both phase transitions & critical end-points $-$ LG & FOPT
  - Phase-diagram at $|\mu_S| \geq 0$; with $\mu_B \geq 0$ & $\mu_I = 0$
  - Properties of ground-state nuclear-matter & neutron-star-matter
  - Quantitative agreement with UrQMD-simulation outcomes
  - Hydrodynamic simulations of HIC's with model EoS's
  - Quantitative agreement with coarse-grain-transport results & HADES data

Lattice-QCD comparison



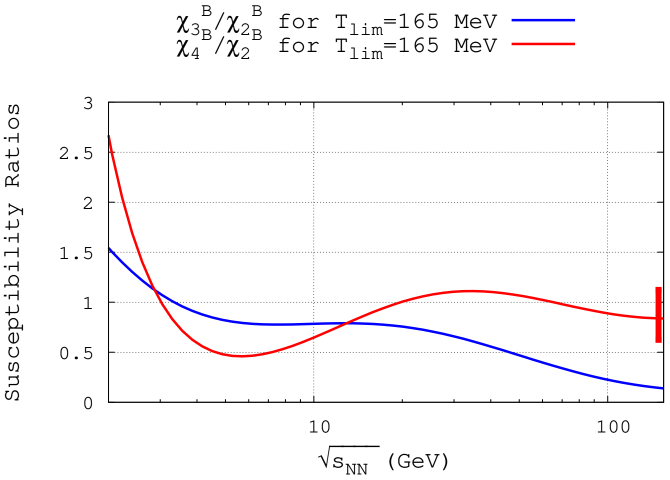
- Mass-degeneracy of parity-doublets at $T > T_{CEP}$ consistent with lQCD

Phase diagrams



- Cumulant-ratios increase near CEP's & near crossover-merger at low μ_B
- Enhancement more pronounce for χ_4^B/χ_2^B

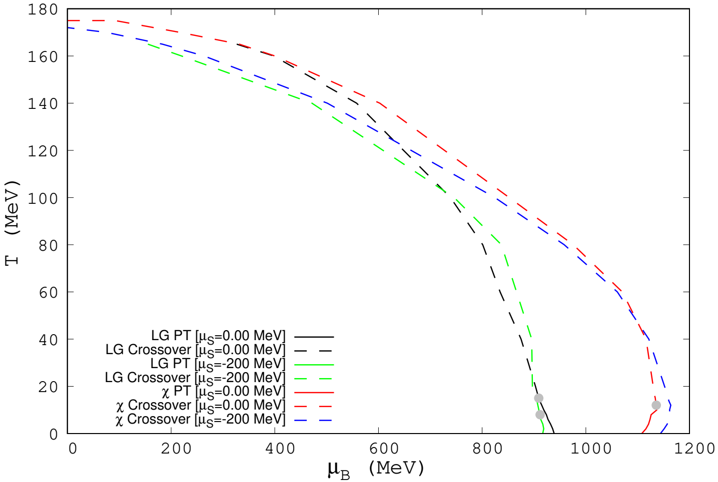
Susceptibilities



[A. Mukherjee, J. Steinheimer & S. Schramm; Phys. Rev. C 96 (2017) 2, 025205]

- Cumulant-ratios deviate considerably from unity

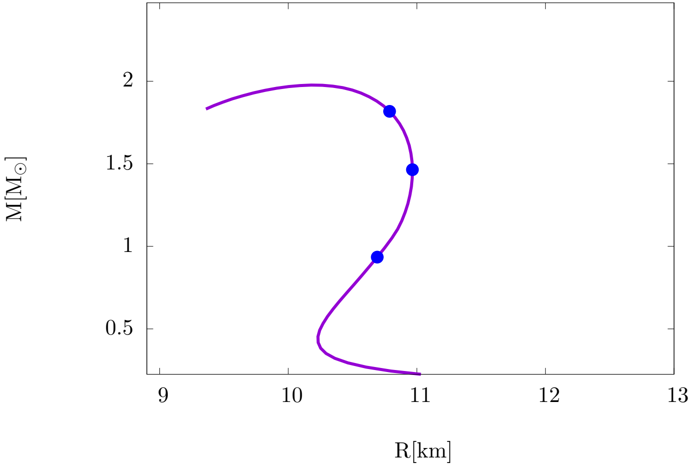
Strangeness



[A. Mukherjee, A. Bhattacharyya & S. Schramm; Phys. Lett. B 797 (2019) 134899]

- Chiral-FOPT disappears for $|\mu_S| \geq 175$ MeV due to hyperon-domination

Astrophysical benchmarks



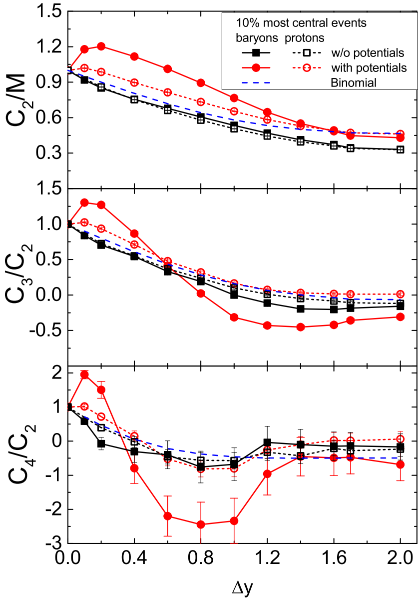
[A. Mukherjee, J. Steinheimer, S. Schramm & V. Dexheimer; Astron. Astrophys. 608 (2017) A110]

- Model-EoS with TOV to generate *M*-*R* diagram for neutron stars
- Maximum mass & radius in agreement with observations

'Numbers' speak louder than words!

- Ground-state nuclear-matter $\kappa = 267.12$ MeV
- Saturation density $(\rho_0) = 0.142$ fm^{-3}
- Binding energy (E/A) or, energy-density per baryon $(\varepsilon/\rho_{\rm B}) = -16$ MeV
- Symmetry energy: $S = \frac{1}{8}\left[\frac{d^2(\varepsilon/\rho_{\rm B})}{d(I_3/B)^2}\right]_{\rho_{\rm B}=\rho_0} = 30.02$ MeV
- Slope parameter: $L = 3\rho_0\left[\frac{dS}{d\rho_{\rm B}}\right]_{\rho_{\rm B}=\rho_0} = 56.86$ MeV
- Maximum star mass: $M_{\rm max} = 1.98\ M_\odot$
- Maximum star radius: $R_{\rm max} = 10.25$ km
- Canonical star mass: $M_{\rm c} = 1.4\ M_\odot$
- Canonical star radius: $R_{\rm c} = 11.10$ km

UrQMD with nuclear potentials



- Enhanced cumulant-ratios with nuclear potentials, for $\Delta y < 0.3$
- For $\Delta y > 0.3$, all C_n suppressed, due to baryon-number conservation
- Enhancement smaller for net-N_p than net-N_B, due to random exchange of isospin with neutrons & pions
- Cascade mode agrees with simple binomial distr. for net-N_B

[J. Steinheimer *et al*; Phys. Lett. B 785 (2018) 40–45]

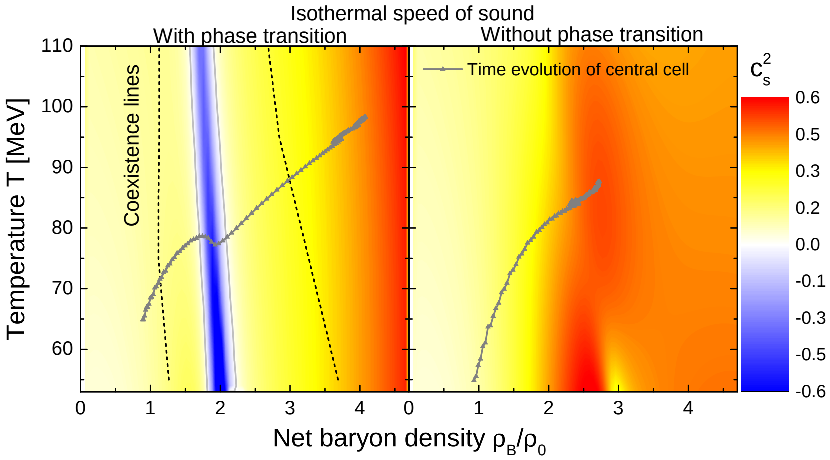
Dileptons

- Dileptons: effective probes for the evolution of the fireball; on account of electro-weak interactions being unlikely at strong-interaction timescales
- Invariant-mass spectrum of dileptons obtained from emissivity

$$\epsilon = -\frac{\alpha_{EM}^2}{\pi^3}\frac{L(M)}{M^2} f^B(q_0; T)\mathrm{Im}\Pi_{EM}(M, q; \mu_B, T)$$

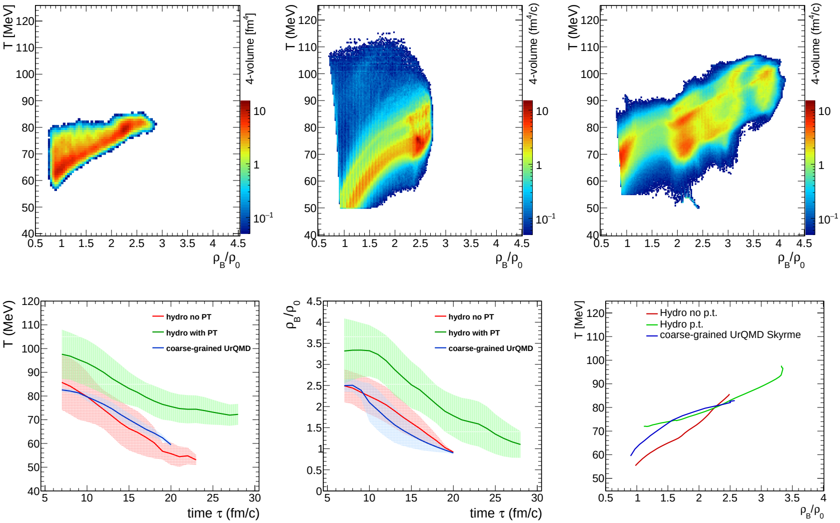
  - M: virtual photon mass = dilepton invariant-mass, M_{ee} $\left(= \sqrt{q_0^2 - q^2}\right)$
  - α_{EM}: electromagnetic coupling constant
  - $\mathrm{Im}\Pi_{\mathrm{EM}}$: EM spectral-function of the QCD medium
  - $f^B(q_0; T)$: thermal Bose distribution
  - $L(M)$: lepton phase-space factor
- **H**igh-**A**cceptance **D**i-**E**lectron **S**pectrometer
- SIS18 BES with Au+Au collisions at 1.23 AGeV measure M through M_{ee}
- Hadronic transport model, using UrQMD
- Hydrodynamic evolution, without first-order phase transition
- Hydrodynamic evolution, with first-order phase transition

Hydrodynamic simulations



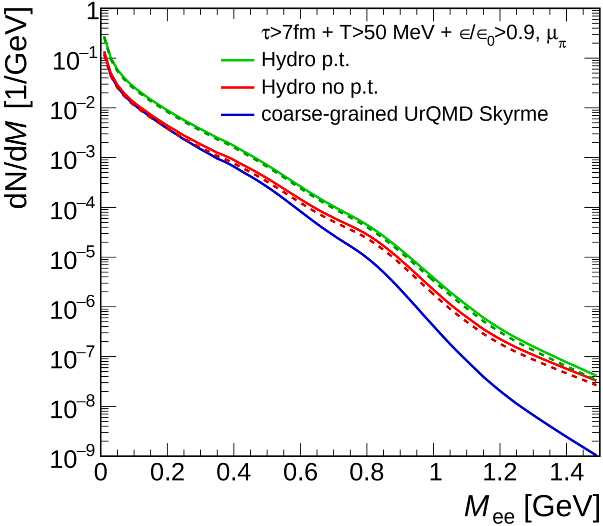
- Two different QχP EoS's as inputs
- Two hydro. sim.'s with three impact parameters – 2 fm, 4 fm & 7 fm
- T & ρ_B obtained as functions of $\mathbf{x}$ and $t \to \epsilon$ and M_{ee}

Bulk evolution



- Higher T & ρ_B and longer system-lifetimes in first-order scenario

Dilepton spectra



[F. Seck *et al*; Phys. Rev. C 106 (2022) 014904]

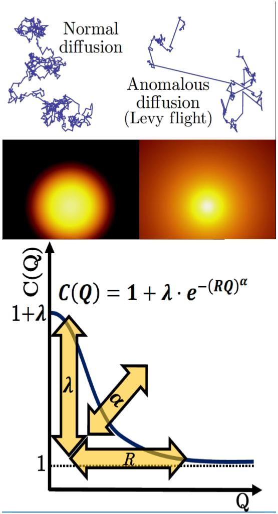
- FOPT doubles low-mass dilepton-yield; over that from crossover; due to prolonged system-lifetime caused by mixed-phase-formation

Outcomes & outlook

- Considerable influence of LG transition on cumulant values
- Acceptable nuclear-matter properties with stiff EoS (excluded volumes)
- Corroborated values for max. mass, canonical mass & canonical radius of NS's
- Double dilepton-yield with FOPT; w.r.t. crossover (prolonged system-lifetime)
- Hadronic re-scattering in dilute phases suppress effects of EoS-driven expansion
- Measurements of initial-state fluctuations, with effective theories
- Model simulations using non-zero net-strangeness and net-isospin
- Dynamic simulations of ultra-relativistic heavy-ion collisions, with UrQMD
- Application of obtained EoS to nuclear-astro. simulations of neutron stars.
- Exploration of similarities between NS-mergers and HIC's.
- Investigations into empirical observables as signatures of critical phenomena
- Probing of finite-size systems, with momentum cut-offs & Matsubara-sums
- Search for exotic phases of matter & hypernuclei, with effective theories
- Examination of magnetic field effects on phases of strongly-interacting matter

PART II: EXPERIMENT

Charged-kaon femtoscopy



- Mapping geometry of source → momentum correlations of like-sign kaon-pairs: $C(q) = 1 + \tilde{D}(q)$; $\tilde{D}(q)$: FT of pair-source $D(r)$
- Usually assumed shape for $D(r)$ – Gaussian
- Generalization – Lévy distribution: $\mathcal{L}(r;\lambda,R) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-(RQ)^{\alpha}} e^{iQr} dQ$
- R: Lévy-scale, λ: correlation-strength, α: Lévy-exponent, Q: integration variable
- $\alpha = 2$: Gauss; $\alpha < 2$: power-law; $\alpha = 1$: Cauchy (or, exponential)
- Possible reasons for non-Gaussian sources:
  - Proximity to CEP: irrelevant at 200 GeV
  - Jet fragmentation: not possible in A+A
  - Anomalous diffusion: viable in A+A at 200 GeV

Hadronic re-scattering

- Evidence of non-Gaussian source-distribution for pions found in Au+Au collisions at PHENIX & STAR
- Extracted coordinate-space distributions show heavy tail
- Hydrodynamic calculations assume idealised freeze-out: sudden jump in mean-free-path from 0 to ∞
- More realistic scenario $-$ hadronic re-scattering:
  - System cools & dilutes with expanding hadron-gas
  - Mean-free-path gradually diverges to ∞, in finite time-interval
  - Re-scattering occurs in time-dependent mean-free-path-system
  - Anomalous diffusion – experimentally observed as power-law-shaped tails in coordinate-space distributions
  - In contrast to Gaussian, strongly-decaying tails for normal diffusion

Normal diffusion

- Momentum-space diffusion equation:

$$\frac{\partial W}{\partial t} = -K_n Q^2 W(Q, t)$$

  - K_n: normal diffusion constant
  - Q: momentum
  - t: time
  - $W(Q, t)$: momentum-space probability distribution

- Coordinate-space solution: $W(r, t) = \frac{1}{\sqrt{4\pi K_n t}} e^{-\frac{r^2}{4K_n t}} \rightarrow$ Gaussian

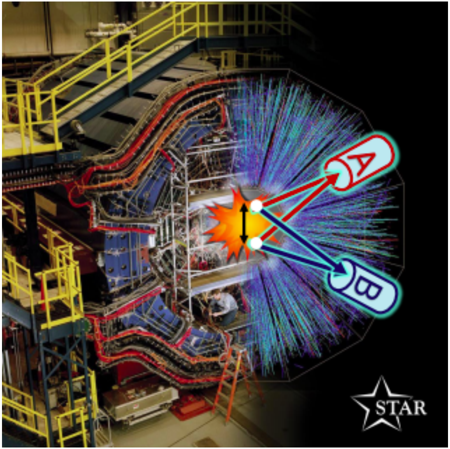
Anomalous diffusion

- Coordinate-space diffusion (generalised Fokker-Planck) equation:
  [T. Csörgő, S. Hegyi, T. Novák & W. Zajc; AIP Conf. Proc. 828 (2006) 1, 525-532]

$$\frac{\partial W}{\partial t} + v\frac{\partial W}{\partial r} + \frac{F(r)}{m}\frac{\partial W}{\partial v} = \eta_{\alpha' 0} D_t^{1-\alpha'} L_{\mathrm{FP}} W(r, v, t)$$

- Momentum-space solution: $W(Q, t) = e^{-tK^{\alpha}|Q|^{\alpha}}$
  - $W(Q, t)$: characteristic function (FT) of Lévy-stable source-distributions
  - α: Lévy-exponent
  - K: anomalous diffusion constant

STAR

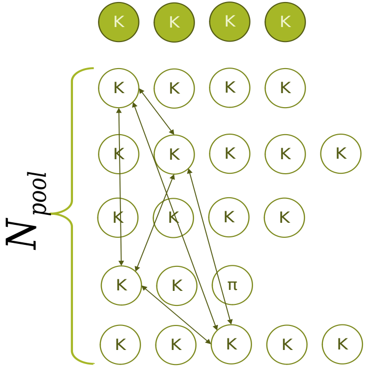


- **S**olenoidal **T**racker **A**t **R**HIC
- Colliding ^{238}U, ^{197}Au, ^{63}Cu, ^{96}Zr, ^{96}Ru, ^{27}Al, ^{3}He, d & p
- Multiple centre-of-mass energies ($\sqrt{s_{\mathrm{NN}}}$) for BES-I & BES-II
- Measurement: RHIC BES (2016) with Au+Au collisions at 200 GeV
- PID: dE/dx for K^+, K^-

Event & track processing

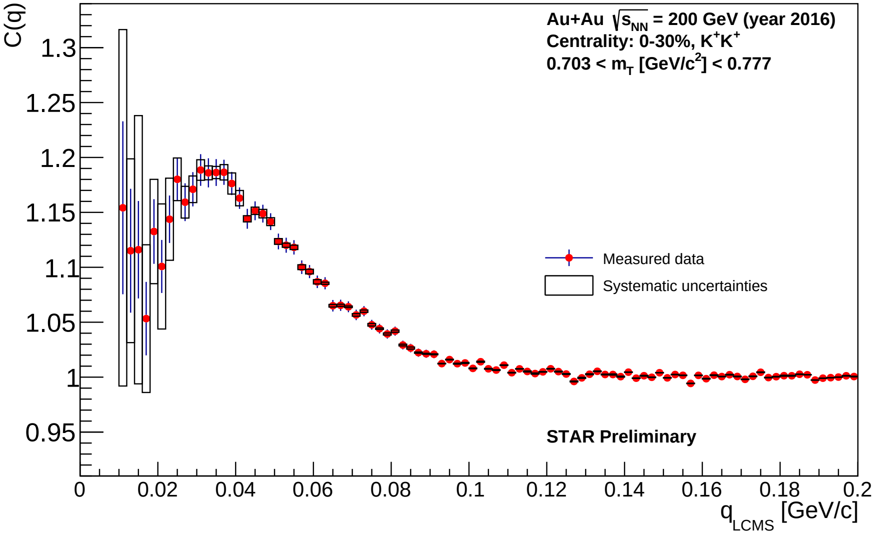
- Event processing:
  - 3.06 billion events from 2016 RHIC beam-energy scan (BES) at 200 GeV in STAR's PicoDST file-storage
  - Trigger cuts (VPD, TPC, etc.) bring no. of events down to 2.59B
  - 0-30% centrality cut further reduces no. of events to 776 million
  - 52.8% of 776M events processed to get particle-tracks for analysis
- Track processing:
  - Tracks read in & cut (PID, N_{Hits}, etc.); $A(q)$ obtained
  - Pair cuts (FMH, SL & $\Delta z - \Delta u$) applied
  - Particles from current event stored in pool; events mixed
  - Over-weighting of events avoided $\rightarrow$ only one particle selected from one event; $B(q)$ & $C(q)$ obtained
  - $C(q)$ fit with Coulomb-corrected Lévy-function
  - Fit parameters extracted & plotted with systematic uncertainties.

Measurement details



- Momentum (q) measured in **L**ongitudinally **C**o-**M**oving **S**ystem: $q_{\mathrm{LCMS}} = |\vec{p}_1 - \vec{p}_2|_{\mathrm{LCMS}}$
- Spherical symmetry in q_{LCMS} ideal for 1D analysis of 3D system
- $A(q)$ - kaon pairs from same event
- $B(q)$ - kaon pairs from mixed event
- Mixed event created by randomly selecting kaon-pairs from pool
- Correlation-function: $C(q) = A(q)/B(q)$
- 3 m_{T} bins used; $m_{\mathrm{T}} = \sqrt{m^2 + (k_{\mathrm{T}}/c)^2}$
- Lévy-type correlation function: $C(q) = 1 + \lambda \cdot e^{-(Rq)^{\alpha}}$

Correlation-function



- Correlation-function shows Bose-Einstein-peak & Coulomb-hole

Lévy-function & Coulomb-correction

- Bowler-Sinyukov formula with Coulomb-repulsion:
  [Y. Sinyukov *et al*; Phys. Lett. B 432 (1998) 248-257]

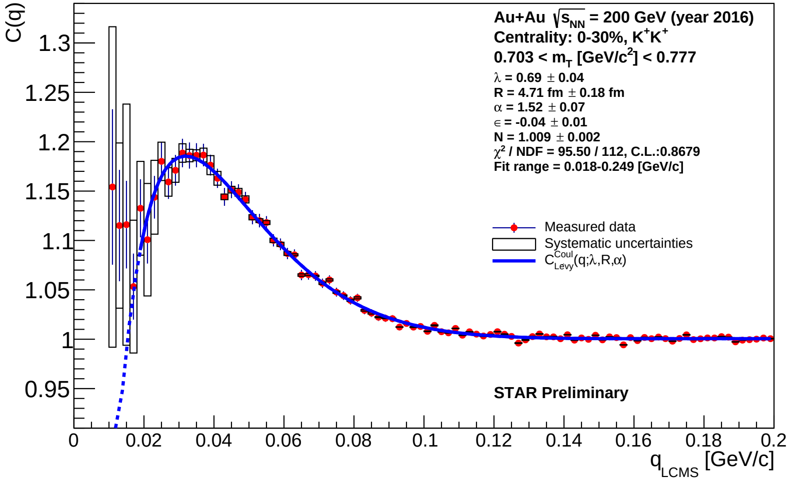
$$C(q) = \left[1 - \lambda + \lambda \cdot K(q) \cdot \left(1 + e^{-(Rq)^{\alpha}}\right)\right] \cdot N \cdot (1 + \varepsilon q) \ ,$$

  - $N \cdot (1 + \varepsilon q)$: assumed linear background
- Coulomb-correction:
  [M. Csanád, S. Lökös & M. Nagy; Phys. Part. Nucl. 51 (2020) 3, 238-242]

$$K(q; \alpha, R) = \frac{\int D(r) |\psi^{\mathrm{Coul}}(r)|^2 dr}{\int D(r) |\psi^{0}(r)|^2 dr} \ ,$$

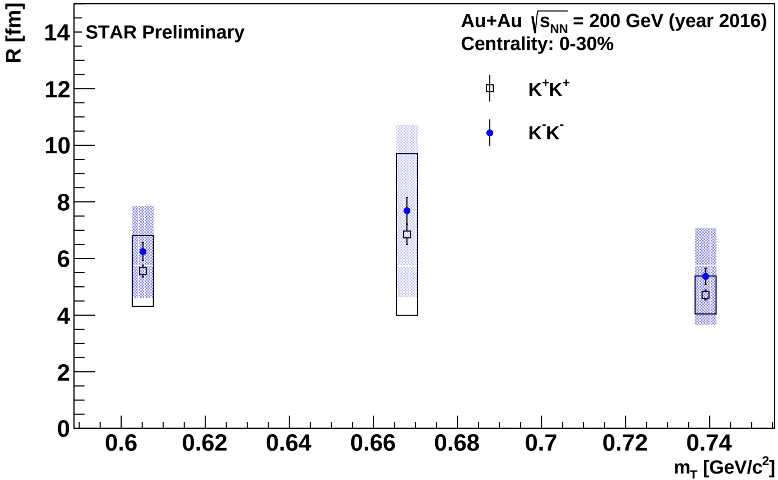
  - $D(r)$: spatial pair-distribution
  - ψ^0: 2-particle plane-wave
  - ψ^{Coul}: Coulomb-wave
  - $K(q; \alpha, R)$ modified for kaons & calculated numerically

Sample fit



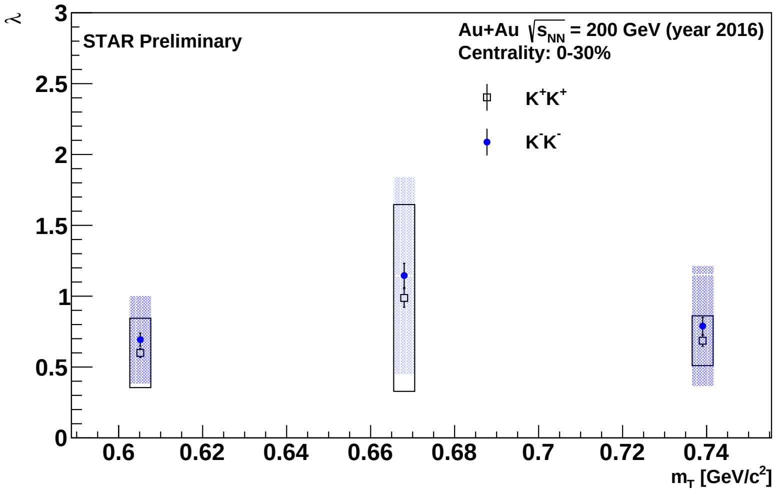
- Measured $C(q)$ agrees quantitatively with best fit over entire q-range
- $N \approx 1$ & $\varepsilon \approx 0$ from fitting – linear contribution negligible

Lévy-scale R



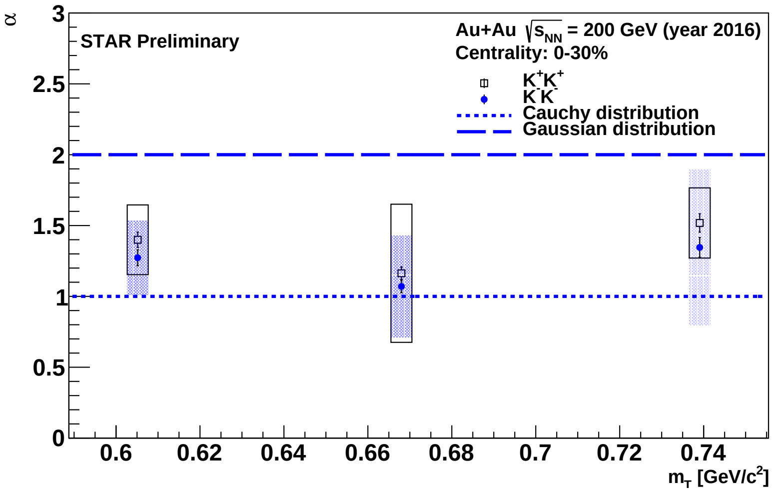
- Kaon-homogeneity length: very weak dependence on m_T; large uncertainties
- Possible slight decrease; not contradicting hydro.-predictions
- Similar to PHENIX pion data: $R_\pi(m_\mathrm{T}{=}0.6{-}0.7\ \mathrm{GeV}/c^2) \approx 5{-}7$ fm
  [PHENIX Collaboration; Phys. Rev. C 97 (2018) 6, 064911]

Correlation-strength λ



- Intercept of correlation-function − Core-Halo model: $\lambda = N_C/(N_C + N_H)$
  [T. Csörgő, B. Lorstad & J. Zimányi; Z. Phys. C 71 (1996) 491-497]
- Close to unity; in line with expected, small fraction of decay-kaons

Lévy-exponent α



- May describe extent of anomalous diffusion
- $\alpha \approx 1.0 - 1.5$ for kaons, similar to PHENIX pion results: $\alpha_\pi \approx 1.2$
  [PHENIX Collaboration; Phys. Rev. C 97 (2018) 6, 064911]
- Suggests non-Gaussian source-shape for charged kaons, similar to pions

Outcomes & outlook

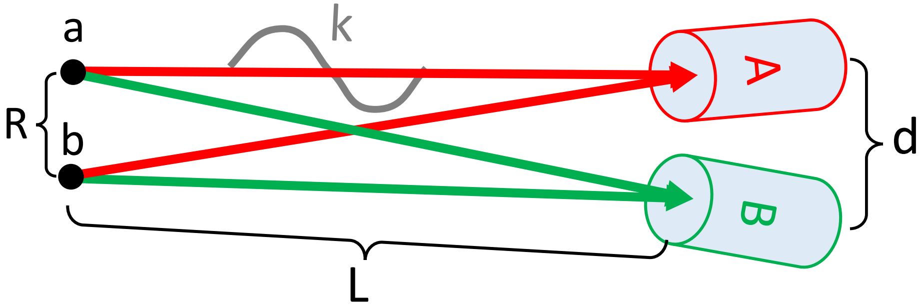
- Preliminary analysis suggests a non-Gaussian source-shape for pairs of charged kaons produced in heavy-ion collisions
- Lévy-stability-exponent α is comparable to that of pions
- Anomalous diffusion is not the sole reason for the heavy tails; since $\alpha_K \approx \alpha_\pi$, instead of $\alpha_K \nless \alpha_\pi$
- Full systematic-uncertainty-analysis is required to achieve definitive conclusions about the geometry of the source
- Similar measurements at lower centre-of-mass energies would be interesting as probes for the critical end-point

Thank you for your attention!

অন্তরে অতৃপ্তি র'বে সাঙ্গ করি' মনে হবে
শেষ হয়ে হইল না শেষ।

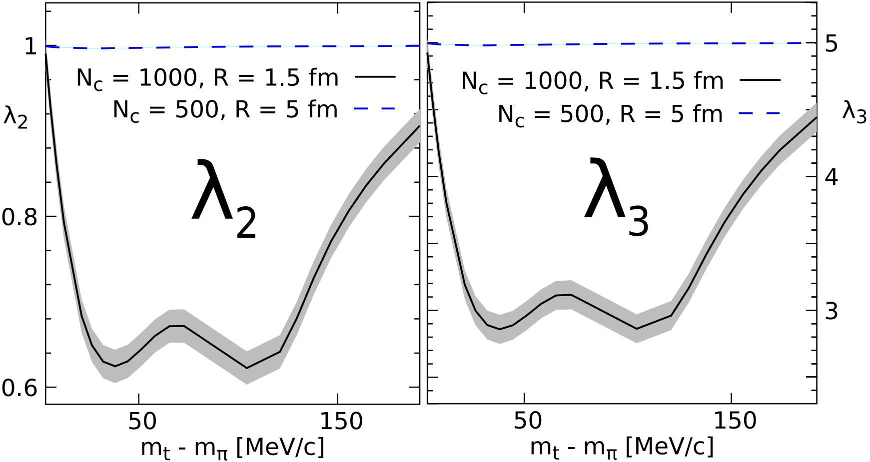
APPENDIX

Random phases



- a and b as sources, A and B as detectors
- R and d as distance between the sources and detectors, respectively
- k as the phase difference and L as the path length
- Two- and three-particle correlation-strengths, with random-phase:
  - $\lambda_2 = C_2(0) - 1 = e^{-2\sigma_\phi^2}$
  - $\lambda_3 = C_3(0) - 1 = 3e^{-2\sigma_\phi^2} + 2e^{-3\sigma_\phi^2}$

Correlation-strengths



[M. Csanad, A. Jakovac, S. Lokos, A. Mukherjee & S. K. Tripathy; Gribov-90 Memorial Volume 7 (2021) 261–273]

- Low-m_t decrease of $\lambda_{2,3}$
- Magnitude strongly depends on charge density