A study of hot and dense strongly interacting systems with the quark-hadron chiral parity-doublet model

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Doctoral Disputation

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HIC for FAIR

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The Basics

Interaction	Theory	Mediator	Rel. Strength	Range (m)
Strong	QCD	Gluons	10^{38}	10^{-15}
Electromag.	QED	Photons	10^{36}	∞
Weak	E-W Theory	W & Z	10^{25}	10^{-8}
Gravity	GR	Gravitons(?)	1	∞

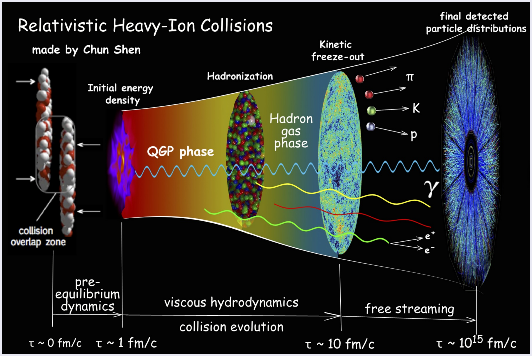
Strong Interaction

- Early universe, neutron stars and heavy-ion collisions (HIC's), all have strong interactions as the dominant force
- HIC's used to recreate the state of matter after big bang and probe the nature of strong interactions
- Quantum Chromodynamics (QCD) details the rules of strong interaction

The Background

HIC & QCD

- Conjectured phases of an HIC



Courtesy: Chun Shen, The Ohio State University

- Analytic insolvability of QCD: the diverging coupling constant

The Background (contd.)

LQCD, PQCD & effective models

- Reason behind the use of LQCD, PQCD & effective model approaches

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4}\sum_{\alpha} F^{\alpha}_{\mu\nu}F^{\mu\nu\alpha} + i\sum_{q}\overline{\psi}^{i}_{q}\gamma^{\mu}\left(D_{\mu}\right)_{ij}\psi^{j}_{q} - \sum_{q} m_q \overline{\psi}^{i}_{q}\psi_{iq}$$

- Effective coupling strength: $\alpha_s \sim 1/\left[\ln\left(Q^2/\Lambda^2\right)\right]$
- Chiral Symmetry: A system which is symmetric in such a way that the system and its mirror-image are super-imposable
- For a non-zero value of the chiral condensate ($\sigma = \langle\overline{q}q\rangle$) the chiral symmetry of $\mathcal{L}_{\mathrm{QCD}}$ is spontaneously broken
- σ as the order-parameter for QCD phase transitions and the consequent divergence of higher-order fluctuation moments ($\chi'\mathrm{s}$) of observables coupled to σ

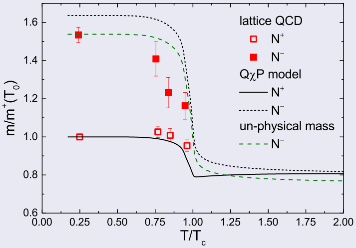
The Model

The Crux

$$\mathcal{L}_{\rm B} = \sum_i (\bar{B}_{\rm i} i \partial\!\!\!/ B_{\rm i}) + \sum_i \left(\bar{B}_{\rm i} m_{\rm i}^{*} B_{\rm i}\right) + \sum_i \left(\bar{B}_{\rm i} \gamma_\mu (g_{\omega {\rm i}} \omega^\mu + g_{\rho {\rm i}} \rho^\mu + g_{\phi {\rm i}} \phi^\mu) B_{\rm i}\right)$$

where

$$m_{\rm i\pm}^{*} = \sqrt{\left[(g_{\sigma {\rm i}}^{(1)} \sigma + g_{\zeta {\rm i}}^{(1)} \zeta)^2 + (m_0 + n_{\rm s} m_{\rm s})^2\right]} \pm g_{\sigma {\rm i}}^{(2)} \sigma \pm g_{\zeta {\rm i}}^{(2)} \zeta$$



$$V = V_0 + \frac{1}{2} k_0 I_2(\sigma, \zeta) - k_1 I_2^2(\sigma, \zeta) - k_2 I_4(\sigma, \zeta) + \mathbf{k_6 I_6}(\sigma, \zeta)$$

The Model (contd.)

Quarks as degrees-of-freedom

Quarks become the dominant degrees-of-freedom post deconfinement transition from hadron gas

Polyakov loop $\Phi = (1/3)\mathrm{Tr}\left[\exp\left(i\int d\tau A_4\right)\right]$, which goes from 0 to 1 during deconfinement, added as order parameter for deconfinement transition

$$\Omega_{\mathrm{q\ or\ \bar{q}}} = -T\sum_{\mathrm{i}\in Q}\frac{\gamma_{\mathrm{i}}}{(2\pi)^3}\int d^3k \ln\left(1+\Phi\exp\frac{E_{\mathrm{i}}^{*}\pm\mu_{\mathrm{i}}}{T}\right)$$

$$\begin{aligned} U &= -\frac{1}{2}a(T)\Phi\Phi^{*} \\ &+ b(T)\ln[1-6\Phi\Phi^{*}+4(\Phi^3\Phi^{*3})-3(\Phi\Phi^{*})^2] \end{aligned}$$

with $a(T) = a_0T^4 + a_1T_0T^3 + a_2T_0^2T^2$, $b(T) = b_3T_0^3T$

Excluded volumes introduced to remove hadrons following deconfinement

The Setup

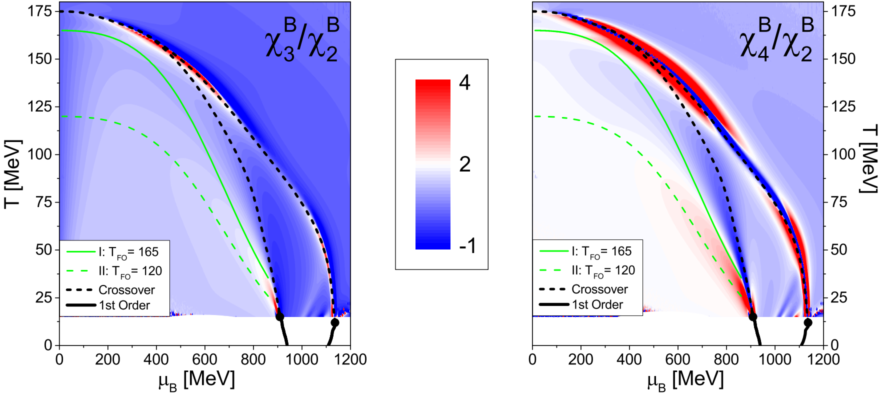
PT lines, freeze-out curves & baryon-number susceptibilities

- PT lines defined as $(\partial\sigma/\partial\mu_{\mathrm{B}})_{\mathrm{max}}$, or as $(\partial\rho_{\mathrm{B}}/\partial\mu_{\mathrm{B}})_{\mathrm{max}}$
- Compressibility: $K(\rho) = 9\rho^2 \frac{\partial^2(E/A)}{\partial\rho^2}\Big|_{\rho=\rho_0}$
- Cumulants or susceptibilities $(\chi_{\mathrm{n}}^{\mathrm{B}})$:

$$\frac{\chi_{\mathrm{n}}^{\mathrm{B}}}{T^2} = n!\ c_{\mathrm{n}}^{\mathrm{B}}(T) = \frac{\partial^{\mathrm{n}}(P(T,\mu_{\mathrm{B}})/T^4)}{\partial(\mu_{\mathrm{B}}/T)^{\mathrm{n}}}$$

- Freeze-out curve, from fit to experimental data, following the Braun-Munzinger prescription, where $\mu_B \sim 1/\sqrt{s_{NN}}$, with $\sqrt{s_{NN}}$ being the beam energy in GeV

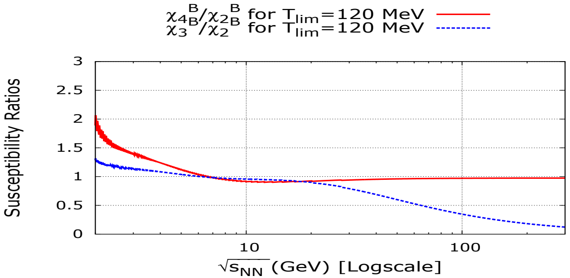
The Outcomes I: Phase Diagrams

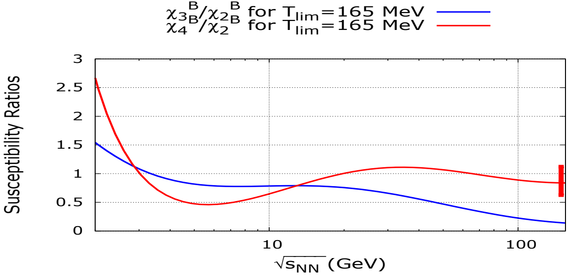


Source: AM, J. Steinheimer & S. Schramm [Phys. Rev. C 96 (2017)]

The Outcomes I: Beam-energy Scans

Shape of you...

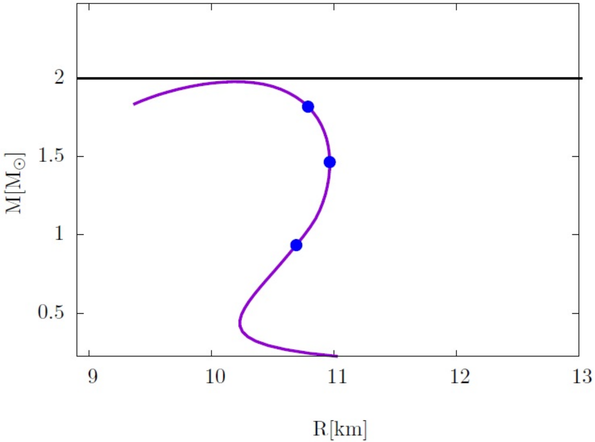




The Application I: Neutron Stars

The QχP EoS & the TOV equations

Complete Equation of State used in the Tolman-Oppenheimer-Volkoff (TOV) equations to generate mass-radius diagram for neutron stars



Source: AM, J. Steinheimer, S. Schramm & V. Dexheimer [Astron. Astrophys. 608 (2017) A110]

The Vindication

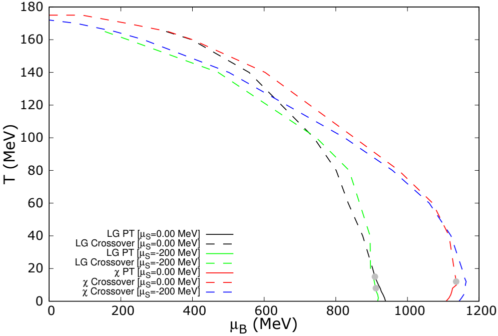
'Numbers' speak louder than words!

- **Ground-state nuclear-matter compressibility (κ) = 267.12 MeV**
- Saturation density (ρ_0) = 0.142 fm^{-3}
- Binding energy (E/A), *a.k.a*:
  Energy density per baryon ($\varepsilon/\rho_{\mathrm{B}}$) = -16 MeV
- **Symmetry energy:** $S = \frac{1}{8}\left[\frac{d^2(\varepsilon/\rho_{\mathrm{B}})}{d(I_3/B)^2}\right]_{\rho_{\mathrm{B}}=\rho_0}$ **= 30.02 MeV**
- **Slope parameter:** $L = 3\rho_0\left[\frac{dS}{d\rho_{\mathrm{B}}}\right]_{\rho_{\mathrm{B}}=\rho_0}$ **= 56.86 MeV**
- **Maximum star mass:** $M_{\mathrm{max}} = 1.98\ M_{\odot}$
- **Maximum star radius:** $R_{\mathrm{max}} = 10.25$ **km**
- Canonical star mass: $M_{\mathrm{c}} = 1.4\ M_{\odot}$
- Canonical star radius: $R_{\mathrm{c}} = 11.10$ km

The Digression: Stranger Things

Non-zero net-strangeness chemical potential

- Local distributions of non-zero net-S could be formed as a result of system fluctuations
- Experimentally, from fitting observed particle ratios, μ_{S} has been deduced to have a value of $\sim 25\% - 30\%$ of μ_{B}

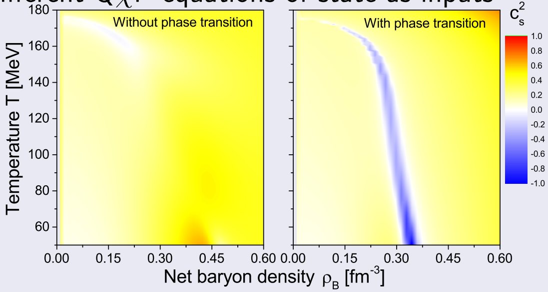


Source: AM, A. Bhattacharyya & S. Schramm (arXiv:1807.11319)

The Application II: Experimental Simulations

HADES

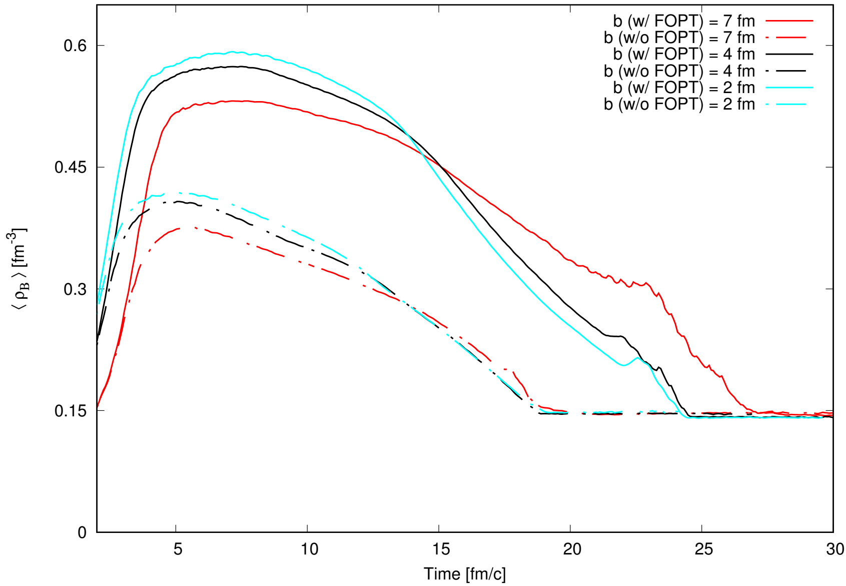
- The invariant mass spectrum of the di-electrons, obtained from the emissivity $\epsilon \equiv f(T, \rho_{\mathrm{B}}, M)$; is measured by the HADES experiment (GSI/SIS18) with beam-energy scans at 1.23 AGeV, using an Au+Au nuclear collision
- Using two different QχP equations of state as inputs



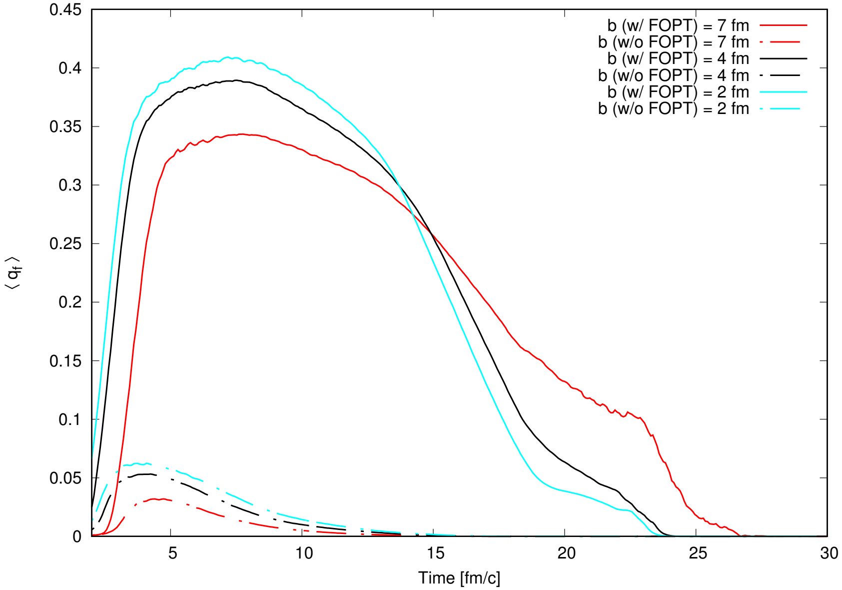
two hydrodynamic simulations are run

- The resulting temperatures, pressures, energy densities, baryon densities and quark fractions are observed

The Outcomes II: Average Baryon Density



The Outcomes II: Average Quark Fraction



The Summary

Conclusions

- Considerable influence of LG transition on cumulant values
- Reasonable similarity to Lattice QCD predictions at $\mu_{\mathrm{B}} = 0$
- Phenomenologically acceptable values for nuclear-matter compressibility, saturation density and energy density per baryon, despite inclusion of excluded-volume corrections which stiffen the EoS
- Successful application of the QχP-generated Equation of State to neutron stars and nuclear matter, as evidenced by the extracted symmetry energy and slope parameter values
- Generation of observationally vindicated values for maximum mass, canonical mass and canonical radius in neutron star family
- Modification of the QCD phase boundary, from a first-order to a smooth crossover, as a result of a non-zero μ_{S}
- Acceptable results for hydrodynamic simulations of HIC's, with the QχP Equations of State

The Outlook

Coming soon... in journals near you

- Comparisons with HADES data: hydrodynamic simulations, focused on di-electron yield and particle-number fluctuations
- Extension of the QχP to finite nuclei
- Effects of isospin-symmetry breaking on the model, and in turn on HIC's
- Magnetic field effects on the QCD phase diagram and fluctuations, using the QχP
- Better agreement between QχP and LQCD calculations
- Tidal deformation calculations for NS's, with the QχP EoS
- Further exploration of the properties of ground-state nuclear-matter inside neutron stars, for different charge fractions

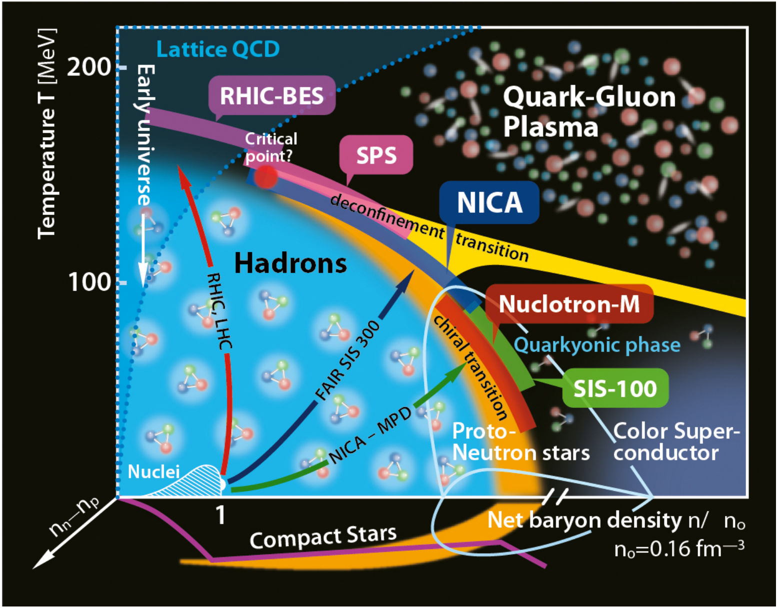
The End

Thank you for your attention!

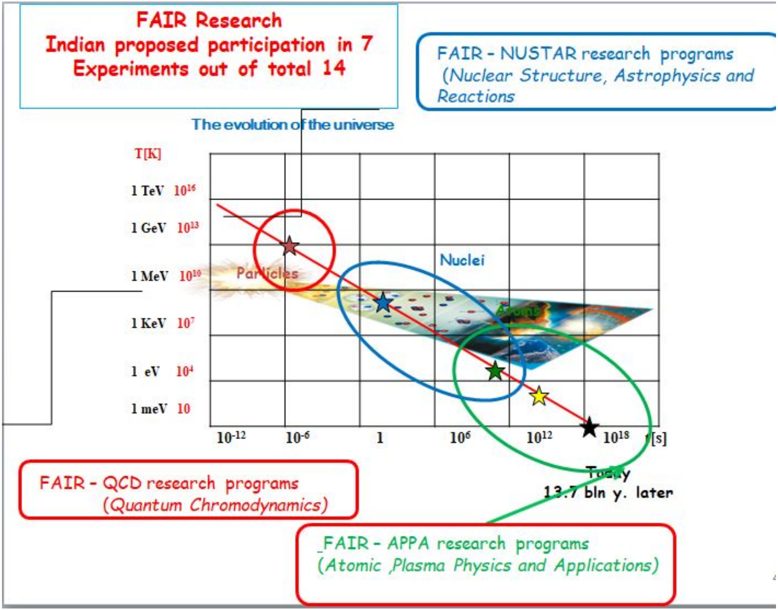
The Backup

অন্তরে অতৃপ্তি রবে সাঙ্গ করি’ মনে হবে
শেষ হয়ে হইল না শেষ।

The Backup: Experiments (Gen.)



The Backup: FAIR (Gen.)



The Backup: TOV, $\mathrm{L_{Q\chi P}}$, Taub & Hydro (Gen.)

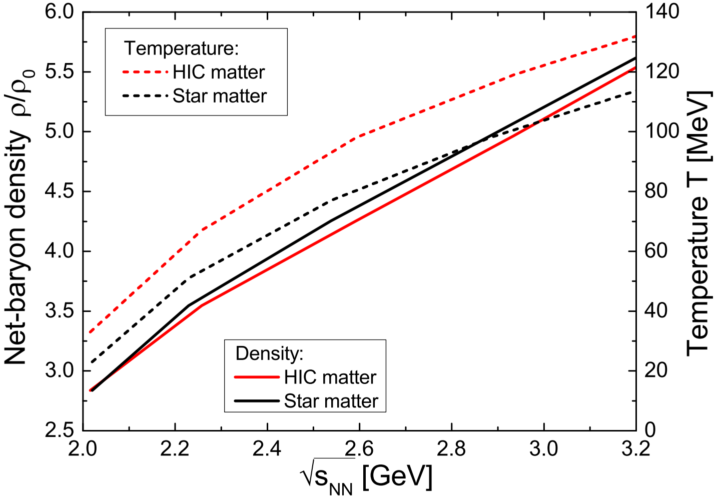
$$\frac{dP}{dr} = -\frac{M\rho}{r^2}\left(1+\frac{P}{\varepsilon}\right)\left(1+\frac{4\pi r^3 P}{M}\right)\left(1-\frac{2M}{r}\right)^{-1}$$

$$L = L_{\mathrm{kinetic}} + L_{\mathrm{interaction}} + L_{\mathrm{meson}}$$

$$(\rho_0 \cdot X_0)^2 - (\rho \cdot X)^2 - (P_0 - P)(X_0 + X) = 0$$

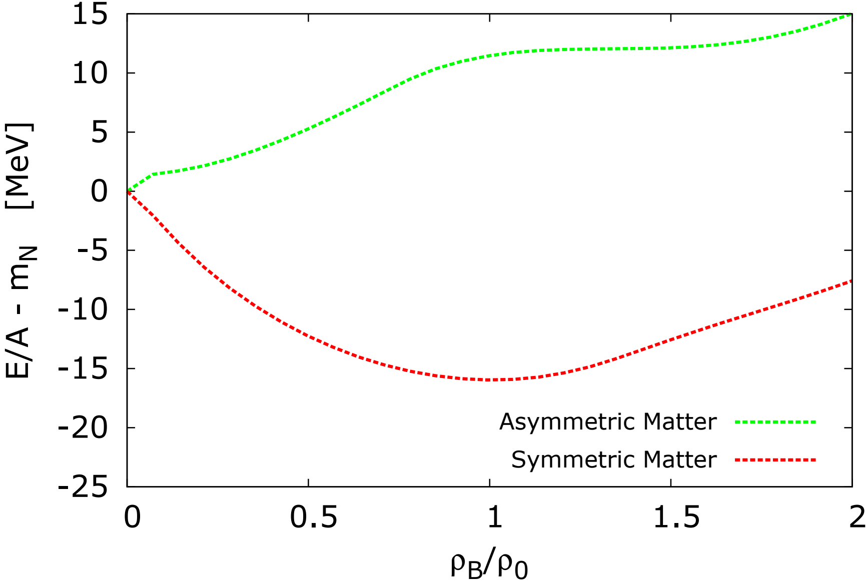
$$\begin{aligned} D\varepsilon + (\varepsilon + P)\theta_\mu u^\mu &= 0 \, , \\ (\varepsilon + P)Du^\alpha + c_s^2 \theta^\alpha \varepsilon &= 0 \, , \\ Dn + n\partial_\mu u^\mu &= 0 \, . \end{aligned}$$

The Backup: Taub Adiabat Calculations (NS)

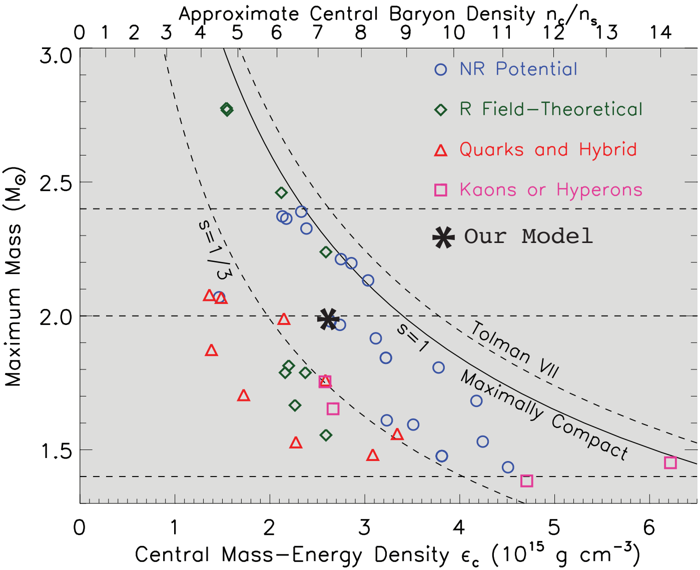


Source: M. Hanauske, AM, H. Stöcker et al J. Phys. Conf. Ser. 878 (2017); J. Steinheimer, AM, H. Stöcker et al Springer Proc. Phys. 208 (2018)

The Backup: Binding energy (NS)

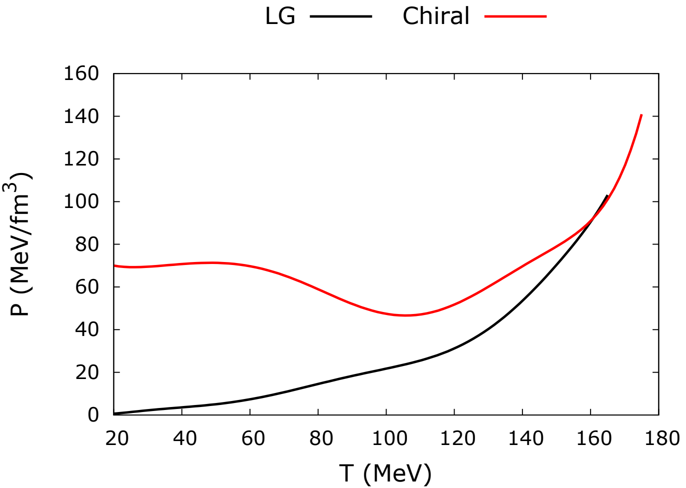


The Backup: Compactness (NS)



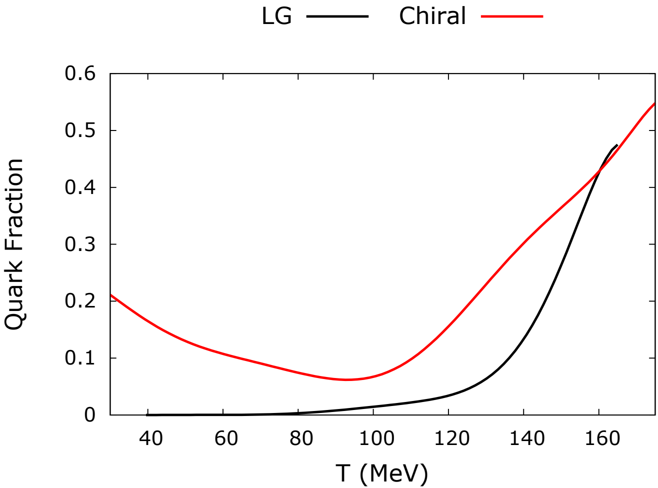
The Backup: Pressure (Mod.)

$$P = -\Omega = (T \ln \mathcal{Z})/V$$

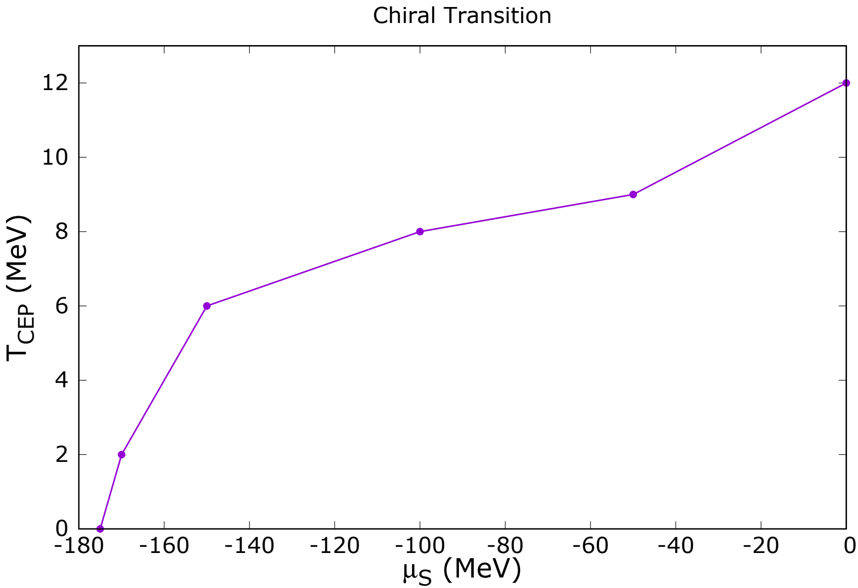


The Backup: Quark fraction (Mod.)

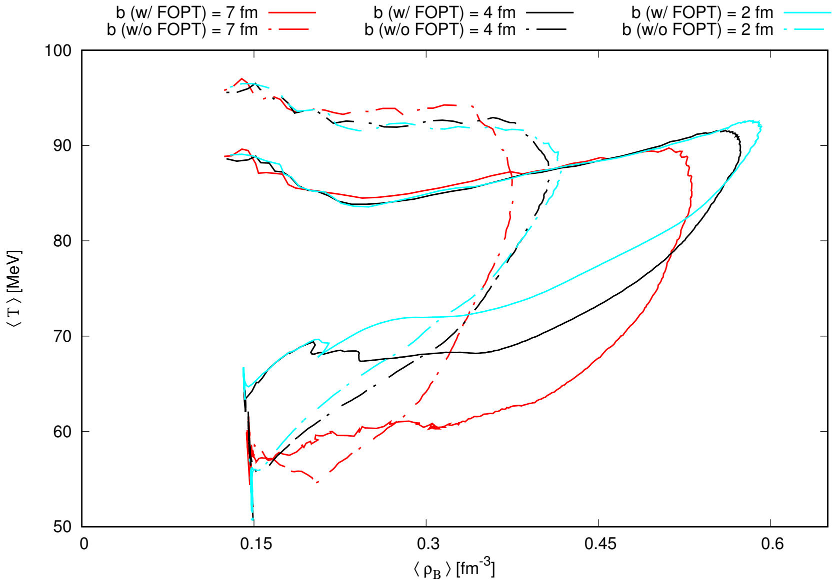
$$q_{\mathrm{f}} = (\varepsilon_{\text{quark}} + \varepsilon_{\text{Polyakov}})/(\varepsilon_{\text{baryon}} + \varepsilon_{\text{meson}} + \varepsilon_{\text{Polyakov}})$$



The Backup: Critical End-point (Str.)



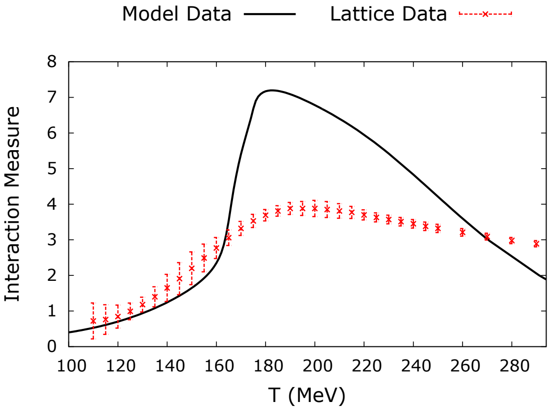
The Backup: Phase-space (GSI)



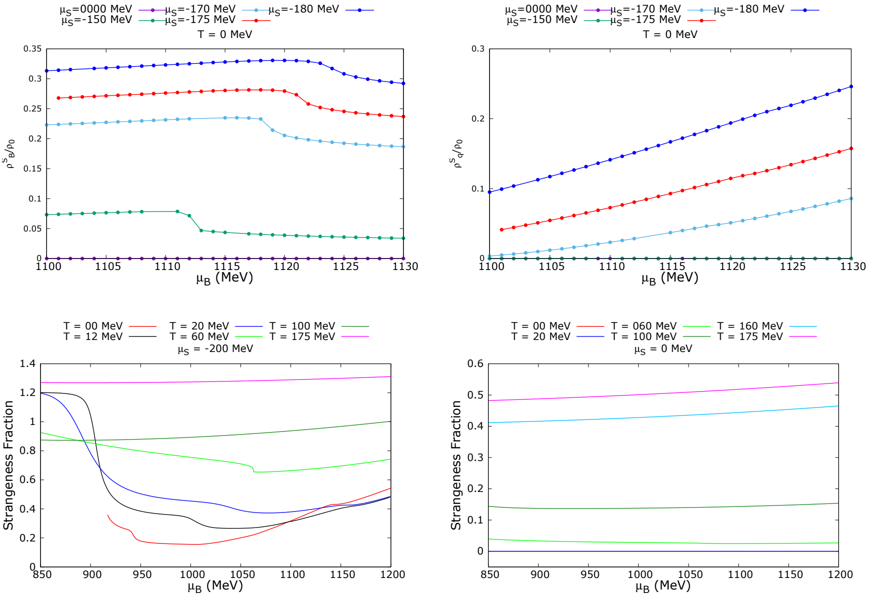
The Backup: Interaction Measure (Mod.)

Lattice data comparison

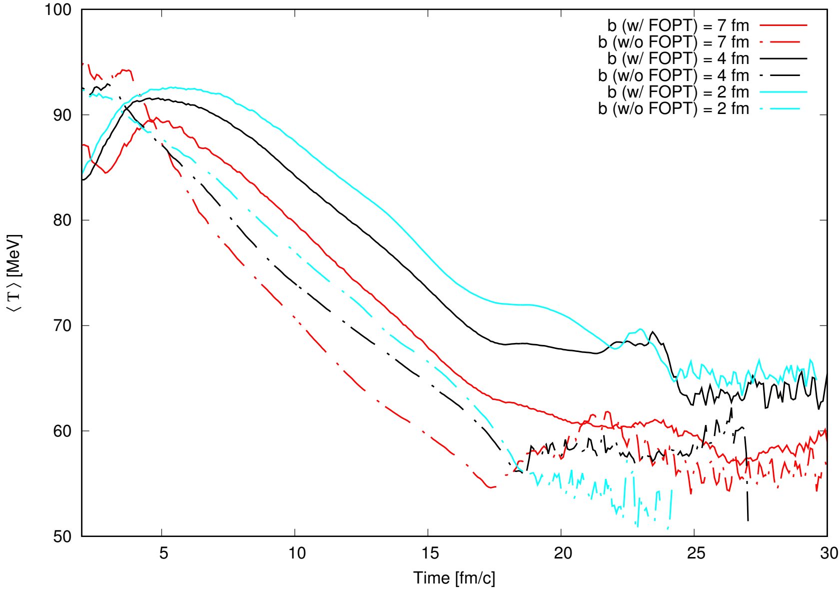
- The model parameters are constrained by actual observables at large ρ_{B} & low T, not by lattice results at $\mu_{\mathrm{B}} = 0$
- Interaction measure, $I = (\varepsilon - 3P)/T^4$, used as means of comparison



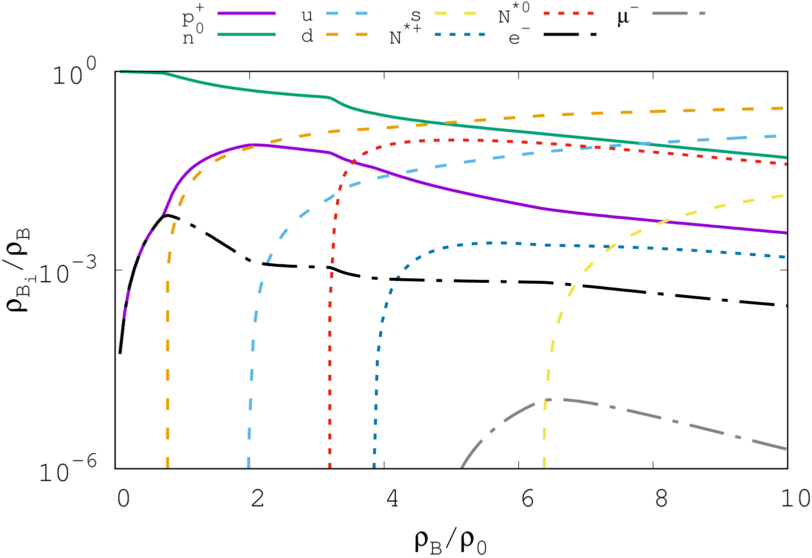
The Hyperon Rises



The Backup: Average Temperature (GSI)



The Backup: Relative Abundance (NS)



The Backup: Non-zero μ_S Chiral PT (Str.)